Technical Notes

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Kernel Function Occurring in Supersonic Unsteady Potential Flow

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Nomenclature

B	$= (M^2 - 1)^{1/2}$
$F_\nu(u)$	= function solution of the nonelementary part of the subsonic Kernel
$I_\nu(u_1,u_2)$	= nonelementary part of the supersonic Kernel
$K(\ )$	= Kernel function relating normal wash at a point x,y,z to a unit pressure difference at a point ξ,η,ζ
$K_\mu(x)$	= modified Bessel function of second kind and order μ
k	= reduced frequency, considered as a constant real nonnegative parameter, $\omega r/U$
M	= Mach number
q	= freestream dynamic pressure, $= \rho U^2/2$
R	$= [x_0^2 + \beta^2 r^2]^{1/2}$, for subsonic flow, and $[x_0^2 - B^2r^2]^{1/2}$, for supersonic flow, $x_0 > Br$
r	$= [(y-\eta)^2 + (z-\zeta)^2]^{1/2}$
U	= freestream velocity in x direction
$w(x,y,z)$	= normal velocity at a point x,y,z
x,y,z	= coordinates of the normal wash point
x_0,y_0,z_0	$= x-\xi,\ y-\eta,\ z-\zeta$
β	$= (1 - M^2)^{1/2}$
$\Gamma(x)$	= Gamma function
$\Delta p(\xi,\eta,\zeta)$	= pressure difference at point ξ,η,ζ
μ	$= 0,1,2,3,\ldots$
ν	$= 1/2,\ 3/2,\ 5/2,\ \ldots, = \mu + 1/\nu$
ρ	= freestream air density
$\psi_\nu(u)$	= functional solution of the supersonic Kernel, $F_\nu(u) + iF'_\nu(u)/k$
$\psi^*_\nu(u)$	= conjugate of the functional solution of the supersonic Kernel, $F_\nu(u) - iF'_\nu(u)/k$
ξ,η,ζ	= coordinates of the doublet point
ω	= frequency of oscillation

Superscript

$'$	$= \mathrm{d}/\mathrm{d}u$

Introduction

THE integral equation relating the pressure and the normal wash distributions in unsteady subsonic potential flow was first derived by Kussner.[1] This equation has the same formal appearance for subsonic and supersonic flows.[2] The difference between the two regimes lies in the associated Kernel function. Many authors[3-6] contributed to the reduction of the subsonic Kernel function to forms suitable for numerical computations. Watkins and Berman[2] extended the analysis of the subsonic formulation to the supersonic case for planar surfaces. Harder and Rodden[7] obtained the supersonic Kernel for nonplanar surfaces.

Exact solutions of the nonelementary part of the subsonic Kernel function have been obtained in Ref. 8, and efficient schemes for the evaluation of these functional solutions have been presented. The purpose of the present paper is to extend the analysis of Ref. 8 to the supersonic case.

Analysis

The integral equation relating the pressure and the normal wash distribution in unsteady potential flows can be written as

$$\frac{w(x,y,z)}{U} = \frac{1}{(8\pi)}\int_s\int \frac{\Delta p(\xi,\eta,\zeta)K(x,y,z,\xi,\eta,\zeta,k,M)}{(qr)^2}\,\mathrm{d}\xi\,\mathrm{d}\eta \tag{1}$$

Equation (1) is valid, whether the flow is subsonic or supersonic. The difference between the two regimes lies in the Kernel function of the integral equation. For both regimes, the nonelementary part of the Kernel can be written as

$$I_\nu(u_1,u_2) = \int_{u1}^{u2}[e^{-ik\nu}/(1+\nu^2)^\nu]\,\mathrm{d}\nu \tag{2}$$

where k is the reduced frequency and is considered a nonnegative real parameter. For planar surfaces $\nu = 3/2$, and for nonplanar surfaces $\nu = 3/2$ and $5/2$. The limits of the integration in Eq. (2) read

Subsonic case

$$u_2 = \infty \tag{3a}$$

$$u_1 = (MR - x_0)/\beta^2 r \tag{3b}$$

Supersonic case

$$u_2 = (x_0 + MR)/B^2 r \tag{4a}$$

$$u_1 = (x_0 - MR)/B^2 r \tag{4b}$$

The subsonic case has been previously treated; it has been shown that the integrals given in Eq. (2) can be evaluated analytically in terms of the function $F_\nu(u)$, given in Ref. 8, where efficient and accurate evaluation of this function has been presented.

Several integral representations of the function $F_\nu(u)$ were further developed in Ref. 9. In the following, an extension of the development of Ref. 8 for the supersonic case is given. In a supersonic regime, the disturbances are restricted to the region of their aft Mach cone, so that x_0 is always positive and $x_0 > Br$. Therefore, in the limits of integrations [Eq. (4)], u_2 is always positive, whereas u_1 can be positive or negative. For values of $x_0 < Br$, there is no disturbance and hence, the Kernel is null. Writing now, for the supersonic case [Eq. (2)] as

$$I_\nu(u_1,u_2) = \int_{u1}^{\infty}\left[\frac{e^{-ik\nu}}{(1+\nu^2)^\nu}\right]\mathrm{d}\nu - \int_{u2}^{\infty}\left[\frac{e^{-ik\nu}}{(1+\nu^2)^\nu}\right]\mathrm{d}\nu \tag{5}$$

Using the development of Ref. 8, and defining

$$\psi_\nu(u) = F_\nu(u) + iF'_\nu(u)/k$$

$$\psi^*_\nu(u) = F_\nu(u) - iF'_\nu(u)/k \tag{6}$$

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after several algebraic manipulations, we obtain

$$I_\nu(u_1,u_2) = ie^{-iku_1}\psi^*_\nu(u_1) - ie^{-iku_2}\psi^*_\nu(u_2), \qquad u_1 > 0 \tag{7}$$

$$I_\nu(u_1,u_2) = [2(\pi)^{1/2}/\Gamma(\nu)](k/2)^\mu k_\mu(k) + ie^{ik|u_1|}\psi_\nu(|u_1|) - ie^{-iku_2}\psi_\nu(u_2), \qquad u_1 < 0 \tag{8}$$

Equations (7) and (8) give a simple and direct way for the evaluation of the nonelementary part of the supersonic Kernel, in terms of the real function $F_\nu(u)$.

Conclusions

Simple and direct expressions for the evaluation of the nonelementary part of the Kernel function of the integral equation relating the pressure and the normal wash distribution in supersonic nonstationary flow has been presented. It has been shown that the solutions presented are related to the same functional solutions of the subsonic Kernel. The expressions presented here can provide the basis for the development of numerical nonstationary interfering lifting surface methods.

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Extensions to the Minimum-State Aeroelastic Modeling Method

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Introduction

In order to account for unsteady aerodynamics in first-order, time-invariant state-space formulation of aeroelastic equations of motion, the aerodynamic forces have to be described as a rational function in the Laplace domain. The Minimum-State (MS) aerodynamic approximation method[1,2] was designed to minimize the number of aerodynamic states in the resulting aeroelastic model. References 2 and 3 applied the MS method to subsonic aeroservoelastic problems with one flutter mechanism and demonstrated a reduction of about 75% of the number of aerodynamic states relative to other methods with the same level of accuracy. The effectiveness of the MS method was increased by the introduction of a physical weighting technique[2] which weights each aerodynamic input data term according to its relative importance. Reference 4 used the MS formulation for additional reduction of the model size by dynamic residualization of high frequency structural states. The MS and the physical weighting procedures are extended in this Note to expand their efficiency and generality and to improve the dynamic residualization. Even though the formulation and numerical examples deal with structural-mode-related aerodynamics only, the extensions are applicable to control surface and gust related aerodynamics as well.

Minimum-State Approximation Procedure

The MS method approximates the Laplace domain generalized aerodynamic force coefficient matrix by:

$$[\tilde{Q}_s(p)] = [A_0] + [A_1]p + [A_2]p^2 + [D](p[I] - [R])^{-1}[E]p \tag{1}$$

where p is the nondimensionalized Laplace variable $p = sb/V$, where b is a reference length and V is the true airspeed. The resulting time-domain state-space aeroelastic equations of motion are presented in Ref. 2. The number of aerodynamic states m is equal to the order to $[R]$.

The input data are unsteady aerodynamic complex matrices $[Q_s(ik_l)] = [F(k_l)] + i[G(k_l)]$, calculated at several $p = ik_l$ points where each $k_l = \omega_l b/V$ is a tabulated reduced frequency. The approximation problem is to find the combination of the real valued $[A_0]$, $[A_1]$, $[A_2]$, $[R]$, $[D]$, and $[E]$ of Eq. (1) that best fit the tabulated data. The $m \times m$ aerodynamic lag matrix $[R]$ is diagonal with distinct negative values to be chosen by the analyst. The applications of Refs. 2 and 4 indicated that the results are not very sensitive to the lag values when they are spread over the range of tabulated k_l values. Three approximation constraints are applied to each term of $[Q_s]$ in order to reduce the problem size by explicitly determining $[A_0]$, $[A_1]$, and $[A_2]$. The formulation of Ref. 2 is extended here to allow more flexibility in constraint selection without increasing the problem size. The three constraints are: 1) data match at $k_l = 0$, which yields

$$A_{0_{ij}} = F_{ij}(0) \tag{2}$$

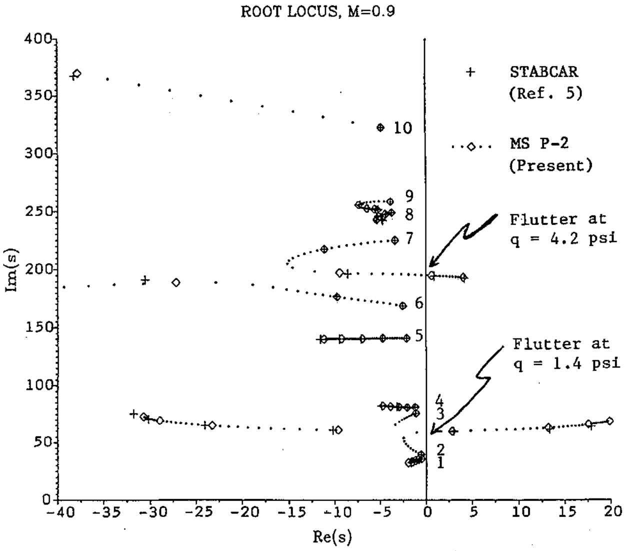

Fig. 1 Comparison of root loci generated by minimum-state and the *p-k* methods, flexible wing at $M = 0.9$.

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